Advanced Training in Mathematics Schools

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Advanced Instructional School in Commutative Algebra

14 May - June 3, 2009

Syllabus

Speaker : Clare D'Cruz, Chennai Mathematical Institute (6 L+ 2 T : Pages 1-19 of [S])

1. **Prime Ideals and Localization:** Nakayama's lemma, localization of rings and modules, Noetherian rings and modules, spectrum of a ring, support of a module, associated primes of rings and modules, primary decomposition of ideals and modules.
2. **Filtrations and graded rings:** Filtered rings and modules, topology defined by a filtration, completions of filtered modules, flatness of completion, graded rings and modules, Artin-Rees lemma, Krull intersection theorem.

Speaker : Balwant Singh, IIT Bombay (8 L + 4 T : Pages 19-50 of [S])

1. **Hilbert-Samuel Polynomials:** Integer valued polynomials and polynomial functions, Hilbert polynomial of a graded module, Hilbert-Samuel polynomial of a filtration.
2. **Dimension of Integral Extensions:** Integral extensions, Cohen-Seidenberg theorems,
3. **Dimension Theorem:** systems of parameters, dimension theorem, normal rings, Krull's theorem.
4. **Dimension theory for Affine algebras :** of polynomial rings, normalization lemma, transcendence degree and dimension, Hilbert's Nullstellensatz, integral closure of a finitely generated algebra.

Speaker : Tony J. Puthenpurakal, IIT Bombay (8 L + 4 T : Pages 266-286 of [M])

1. **Homology of complexes** : Chain complexes, long exact sequence of homology.
2. **Derived Functors** : Projective and injective modules, Derived functors, Ext, Tor.
3. **Koszul Complex** : Tensor product of complexes, Koszul complex, depth and Koszul homology.

Speaker : J. K. Verma, IIT Bombay (4 L+ 2 T : Pages 3-18 of [BH])

1. **Depth of a module:** Regular sequences, Ext and grade, depth and projective dimension, Auslander-Buchsbaum formula.
2. **Cohen-Macaulay modules :** preservation of Cohen-Macaulay property, unmixed theorems of Macaulay and Cohen, CM rings are universally catenary.

Speaker: A. V. Jayanthan, IIT Madras (4 L+ 2 T: Pages 65-72 of [BH]+Pages 31-33 of [H])

1. **Regular local rings :** Auslander-Buchsbaum-Serre criterion, localizations and factorial property of of regular local rings,
2. **Jacobian Criterion :** Singular and nonsingular points of affine varieties, closedness of singular locus.

Speaker : K. N. Raghavan, IMSc, Chennai (6 L+3 T : Pages 88-106 of [BH])

1. **Matlis Duality**: Essential extensions, injective hull, basic properties of injective modules, Matlis duality,
2. **Gorenstein Rings :** Ext and injective dimension, Gorenstein rings, zero-dimensional Gorenstein rings.
3. **Examples :** Of higher dimensional Gorenstein rings

Speaker : R. C. Cowsik, Mumbai University (Expository lectures)

Unity of Mathematics Lectures : Three lectures on Stanley's Upper Bound Theorem on Spheres

L= A lecture of 90 minutes duration T=A two hour tutorial

References

[BH] W. Bruns and J. Herzog, Cohen-Macaulay Rings, Revised Edition, Cambridge, 1998.

[H] R. Hartshorne, Algebraic Geometry, Springer, 1977.

[M] H. Matsumura, Commutative Ring Theory, Cambridge, 1986

[S] J. P. Serre, Local Algebra, Springer 1999.